and at super-satellite speeds, using a correlation in terms of the wall temperature, the product $\rho_w \mu_w \beta$, and the freestream velocity. This correlation holds not only for air but also for $CO_2$.

It is interesting to note that a similar dependency has been shown previously,[2, 3] where the correlation was based on the results obtained from an integral method at temperatures up to 8500°K and did not include the region of large ionization in the boundary layer. The results from this analysis were divided into two cases:

*Case 1:*

$$0.5 \leqslant h_w/h_0 \leqslant 1.0 \qquad Nu/Re_x^{1/2} = 0.744\, Pr_w^{0.656} \tag{1}$$

*Case 2:*

$$h_w/h_0 < 0.5 \qquad Nu/Re_x^{1/2} = 0.848\, Pr_w^{0.656}(h_w/h_0)^{0.154}$$

The correlation of Hoshizaki is based on results that lie mainly in the range of case 2. The expression for the heat transfer rate for case 2 which is arrived at in Ref. 2 is

$$(\dot{q}_w = 0.349 \times 10^4[(h_w)^{0.154}/(Pr_w)^{0.344}](\rho_w\mu_w\beta)^{0.5} (V_\infty/10^4)^{1.692}[1 - (h_w/h_0)] \tag{2}$$

where it is assumed that $h_0 = \frac{1}{2}V_\infty^2$. For $T_w = 520°$R and $Pr_w = 0.72$, Eq. (2) reduces to

$$\dot{q}_w = 3.91 \times 10^4(\rho_w\mu_w\beta)^{0.5}(V_\infty/10^4)^{1.692}[1 - (h_w/h_0)] \tag{3}$$

This is almost identical with Ref. 1 except for the constant. (Hoshizaki's value for this condition is $3.88 \times 10^4$.)

It should be noted that the results of Ref. 3 indicate that the exponent on the freestream velocity is 2 for the adiabatic wall case, then gradually decreases until $h_w/h_0 = 0.5$ where the exponent is 1.692, and then remains constant as the wall becomes relatively colder.

It should be acknowledged that the results of Hoshizaki show the consistency of boundary layer behavior even when complex real gas phenomena are taking place. It is apparent also that the expressions for the basic heat transfer parameter, Eq. (1), may be extended beyond the range over which the correlation was obtained and considered a general expression. The limits on Eq. (1) may be determined experimentally only.


### References

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# Corrections and Comments on "Aerodynamic Processes in the Downwash Impingement Problem"


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It was shown in a recent paper[1] that one mechanism causing particle entrainment in a jet impinging on the ground stemmed from the lift force on the particle. The lift force

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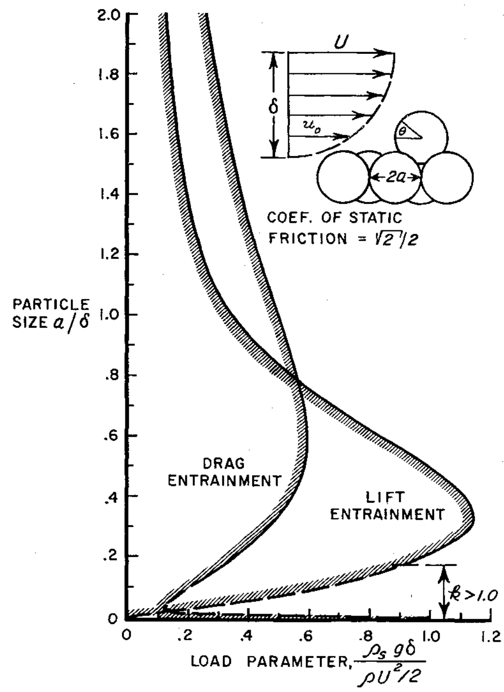


**Fig. 1 Criteria for particle entrainment in an axisymmetric stagnation flow, $u/u_0 \approx R/D$**

was regarded there as the sum of two components, the interference lift due to wall proximity and the lift stemming from the stream shear, and was estimated using existing theoretical solutions. The two components were found to be about equal in magnitude.

It has been brought to the author's attention that an error in sign was made in calculating the interference lift [Eq. (7) of Ref. 1],† and that the classical solutions for a sphere under the influence of a wall show that the body is attracted toward the wall. The purpose of this note is to take cognizance of that error, to show that the interference lift on a sphere close to a wall is small in comparison with the lift due to shear, and to present new criteria for particle entrainment based on more accurate estimates of the particle forces.

The classical approach in calculating the interference effects of a wall on a sphere[2, 3] is to replace the wall with an array of sphere images. An expansion procedure is employed in which the sphere radius is assumed small in comparison with the distance from the wall to the sphere center. Consequently, these solutions cease to be valid in the present problem, where the sphere is in contact with the wall.

There are experimental data[4] obtained with the sphere center 1.08 radii from a wall and with various support structures between the wall and the sphere. With no structure between the sphere and wall, the lift coefficient was −0.03. This compares with $C_L = -0.38$ predicted by interference theory.[2, 3] This measured lift due to wall interference is small in comparison with the lift due to stream shear under typical conditions, and it is concluded that the wall interference effect on lift is negligibly small in the present problem.

These results[4] demonstrate an additional lift-producing mechanism in the particle-entrainment problem which has not been considered. Data were obtained with four thin support columns between the sphere and the wall. The net result of these columns and their wakes was to eliminate almost completely the suction pressure over the enclosed area. The resulting lift coefficient (away from the wall) was 0.30. When the support columns were enclosed completely with a collar,

† The author would like to express his appreciation to E. Levinsky of the Aerospace Corporation for pointing out this error.

the enclosed region was at ambient pressure and the observed lift coefficient was 0.41.

The model originally used to eliminate the particle forces (Fig. 1) was a single sphere resting on a bed of equal-size spheres. In view of the forementioned data, it is apparent that the pressure on the surface in contact with the bed will be near the ambient-stream static pressure. Since the gross interference effects of the wall are negligible, the particle forces can be estimated using the solution for a sphere in a nonuniform flow. The method used is to assume that solution for the pressure distribution on a sphere in uniform shear flow[5] applies up to the point where the sphere contacts the bed, and to assume that the pressure on the surface in contact with the bed is the ambient static pressure. For $k = (a/u_0) \times (\partial u/\partial y)$, the tangential velocity on the sphere is, to the second order in $k$,

$$v/u_0 = \tfrac{3}{2} \sin\theta + k(2 \sin^2\theta - 1.3505) + 0.967k^2 \sin\theta + \ldots \quad (1)$$

The lift on the sphere follows directly from the foregoing assumptions:

$$C_L = 6k \left\{ \left(\frac{8}{5} - A\right)\left[1 - \cos\theta_1 - \frac{1}{3}(1 - \cos^3\theta_1)\right] - \frac{2}{5}\sin^4\theta_1 \cos\theta_1 \right\} + \frac{2}{\pi}\int_{\theta_1}^{\pi/2} \left\{ 2(B \sin^2\theta - 1) \times (\sin^2\theta - \sin^2\theta_1)^{1/2} \sin\theta + 3k(2\sin^2\theta - A) \times \left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\sin\theta_1}{\sin\theta}\right) - \frac{\sin\theta_1}{\sin^2\theta}(\sin^2\theta - \sin^2\theta_1)^{1/2}\right] \sin^3\theta + 2k^2 \sin\theta (\sin^2\theta - \sin^2\theta_1)^{1/2} \left[\frac{2}{3} + \frac{1}{3}\frac{\sin^2\theta_1}{\sin^2\theta}\right] \times [A^2 - (4A - C)\sin^2\theta + 4\sin^4\theta] \right\} d\theta \quad (2)$$

where $A = 1.3505$, $B = 2.25$, $C = 2.901$, and $\theta_1$ is the absolute magnitude of $\theta$ where the sphere contacts the bed.

There are two important items to be noted in Eq. (2). The limit of $\theta_1 = \pi/2$ corresponds to the sphere in uniform shear flow and yields a lift coefficient, $C_L = 0.998k$. This value differs from that given in Ref. 1 since the latter was obtained with a stripwise integration. Second, it should be noted that there is no second-order effect of shear on the lift owing to the symmetry in Eq. (1).

Equation (2) has been integrated numerically for the model illustrated in Fig. 1, i.e., for $\theta_1 = 55°$. The resulting lift coefficient is

$$C_L = 0.345 + 0.557k + 0.879k^2 \quad (3)$$

This result has been applied following the method described in Ref. 1 to arrive at approximate criteria for particle entrainment. The one important difference in this computation is to note that one must define the position of an equivalent smooth wall. The wall position arbitrarily is assumed to be where the sphere contacts the bed. The resulting criteria for particle entrainment are given in Fig. 1. The interpretation of these results is identical to that for Fig. 7 in Ref. 1. The important item to be noted is that lift entrainment is predicted for a wider range of particle sizes and flow conditions than in Ref. 1. Similarly, drag entrainment is predicted for a smaller range of conditions, owing to the assumption as to the effective position of the boundary layer base. Included in Fig. 1 are the particle sizes for which the shear $k$ exceeds unity. For $k \geq 1$, the theory of Ref. 5 ceases to be valid.

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# Errata and Addendum: "A Second-Order Theory of Entry Mechanics into a Planetary Atmosphere"


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Reference 1 was published when the author was in Bulgaria to attend the XIIIth International Congress of Astronautics. Because of lack of author's proofreading, several printing errors were found in the paper. Therefore, the following errata should be made immediately.

## I. Mismatch between Fig. Nos. and Fig. Captions

The figures were renumbered by the editorial staff, whereas the captions corresponding to each originally numbered figure were retained by the printer. This results in 1) mismatch between the figure and the caption in each figure, and 2) mismatch between the figure number with the figure number referred to in the main text.

In order to match the right figure with the right caption and to match the right figure number with the figure number referred to in the main text, the following errata should be made:

1) Figures 7c and 7d: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$) and for $L/D = 1.0$" should be corrected to read as Figs. 3a and 3b: "Comparison of complete spectrum of $L/D$ gliding entry at small initial angles of inclination ($\theta_f \cong 0$)."

2) Figures 3a and 3b: "Comparison of complete spectrum of $L/D$ gliding entry at small initial angles of inclination ($\theta_f \cong 0$)" should be corrected to read as Figs. 4a and 4b: "Comparison of ballistic entry at small and large initial angles of inclination $\theta_f$."

3) Figures 4a and 4b: "Comparison of ballistic entry at small and large initial angles of inclination $\theta_f$" should be corrected to read as Figs. 5a and 5b: "Comparison of the effect of $W/(C_DA)$ ten times larger and ten times smaller."

4) Figures 5a and 5b: Comparison of the effect of $W/(C_DA)$ ten times larger and ten times smaller" should be corrected to read as Figs. 6a and 6b: "Comparison of the effect of $\beta R_0$ ten times larger and ten times smaller."

5) Figures 6a and 6b: "Comparison of the effect of $\beta R_0$ ten times larger and ten times smaller" should be corrected to read as Figs. 7a and 7b: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)."

6) Figures 7a and 7b: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)" should be corrected to read as Figs. 8a and 8b: "Comparison of supercircular-velocity entry at large initial angle of inclination ($\theta_f = 12°$)."

7) Figures 8a and 8b: "Comparison of supercircular-velocity entry at large initial angle of inclination" should be corrected to read as Figs. 7c and 7d: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)."

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